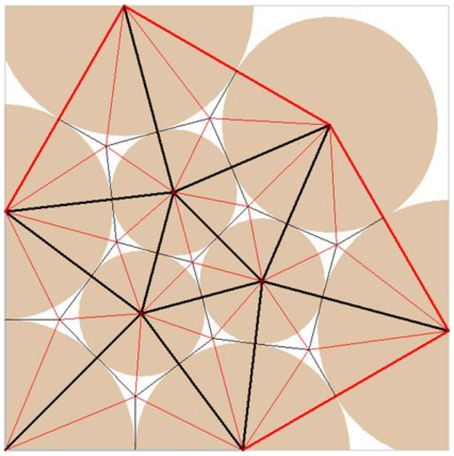
Origami Flower Design

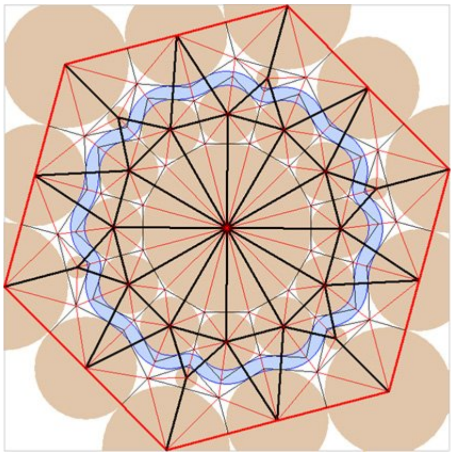
by Derek McGann

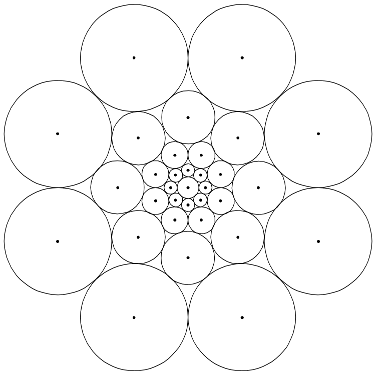
The art of origami originated in the ancient Japanese culture which was dominated by the Samurai, but it was not until modern mathematicians developed an interest in the craft that it truly began to blossom. While learning the folding techniques and, later, the design processes, my interest in the art was dominated by a few categories – insects, sea creatures, flowers, and birds. Of these categories, my primary interest has been in the area of origami flowers. As I began creating my own original designs for origami flowers, a natural starting point was to study pictures of the real things. This study led me to a conclusion which has dominated my flower designs – the petals of many flowers are arranged in concentric circles which are at a slight rotational off-set from each other. Look at the following picture of an iris, the structure of which most clearly demonstrates this principle.

In this picture, there are three layers of petals. The center layer is the smallest. The next layer is not only larger, but is also off-set by 60° in relation to the center layer. The outer layer is larger still, and is also off-set by 60° from the middle layer, making it line up with the center layer. Because of this, I designed an origami iris whose crease pattern is shown to the right.

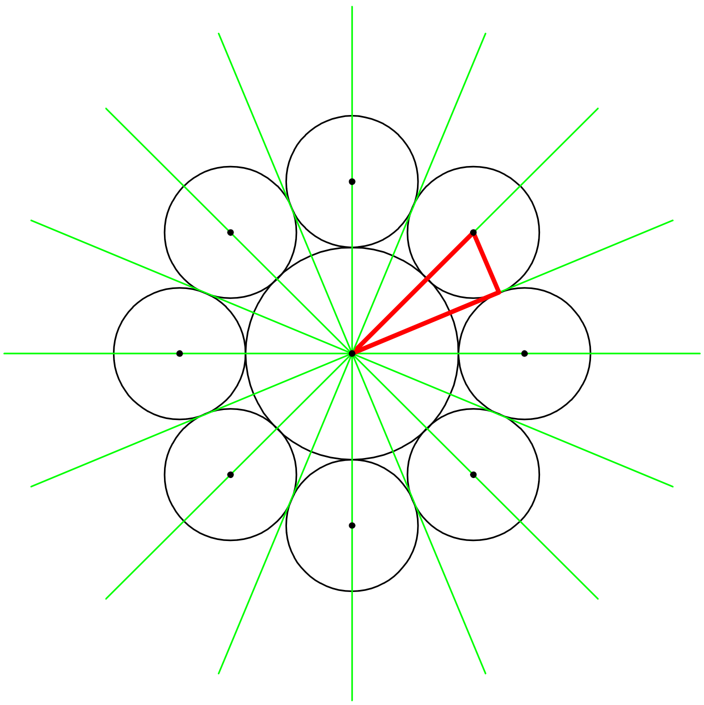


This approach to origami flower design has led to an increased number of both layers and number of petals per layer. After the iris, I designed a torch ginger. To the right are pictures of my finished torch ginger and its crease pattern. This model varies from the basic concept based on the fact that the second ring only has six petals, while the first ring has twelve. After the torch ginger, I designed a water lily. This design had even more petals and layers of petals. For this model, I needed the arrangement of circles shown to the right. This time, however, I decided to calculate the size and position of the outer rings based solely on the radius of the center circle, instead of arranging the circles freehand. Although this model has four rings of eight petals, this approach can easily be altered to any different number of layers and petals per layer.





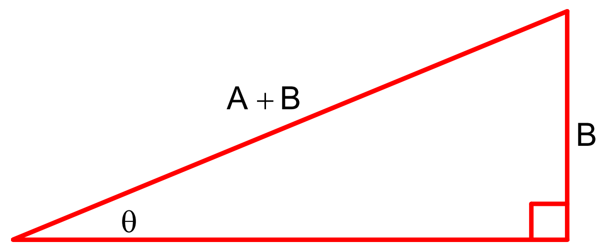
The radius of the center circle, as well as the central angle, can be used to calculate the radii of each of the other circles as well as their positions. Each of the circles in the first ring needs to be tangent to the center circle as well as tangent to each of the circles to either side of it in the ring. Using the red triangle, the radius of the circles in the first ring, B, can be calculated as a function of the radius of the center circle, A, and the central angle, θ. In this case, there is no need to separately calculate the position of these circles – the distance of the center of the new circle from the intersection of the ray and the old circle is the same as the radius of the new circle.



$A =$ The radius of the center circle

$B =$ The radius of the circles in the first ring

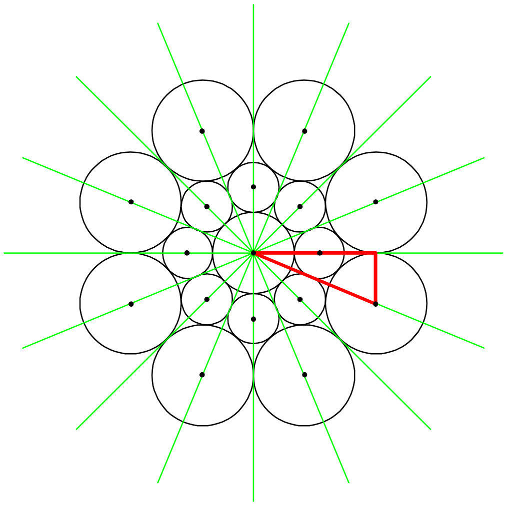
$\theta =$ The central angle (in this case, $\theta = 22.5°$)

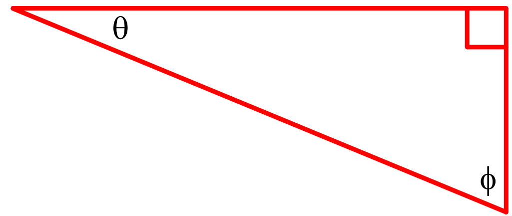


$$\sin(\theta) = \frac{B}{A+B}$$
$$(A+B)\sin(\theta) = B$$
$$A\sin(\theta) + B\sin(\theta) = B$$
$$A\sin(\theta) = B - B\sin(\theta)$$
$$A\sin(\theta) = B(1-\sin(\theta))$$
$$B = \frac{A\sin(\theta)}{1-\sin(\theta)}$$

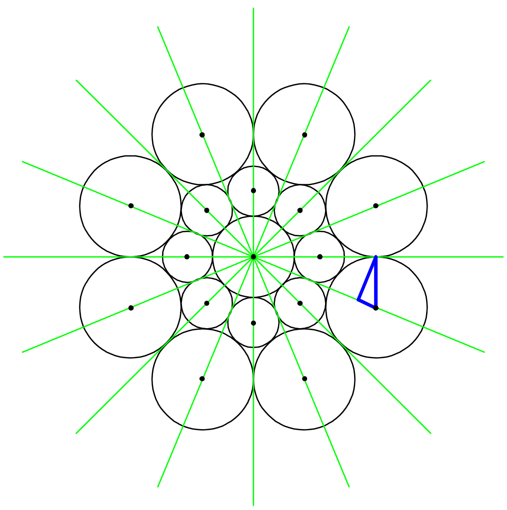
Now that the radius for each of the circles in the first ring is known as a function of A and θ, this value can also be used to determine the size and position of the circles in the next ring. The second ring is not as easy to calculate. Each circle in this ring must be tangent to four other circles – two circles from the first ring as well as the circles to either side from the second ring. The goal is, once again, to determine the radius of each circle in the second ring, as well as their position, as functions of the radius of the center circle and the central angle.

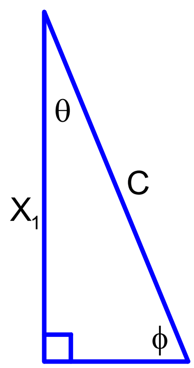
$\phi = 90 - \theta$ (in this case, $\phi = 67.5^\circ$)





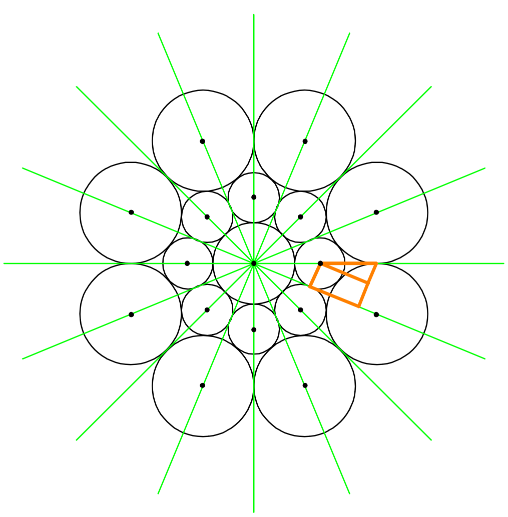
C = The radius of each of the circles in the second ring

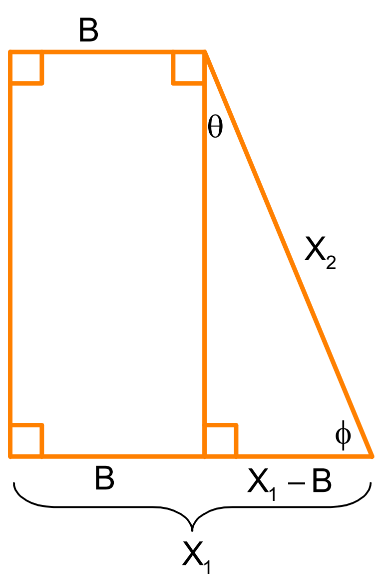




$$\cos(\theta) = \frac{X_1}{C}$$

$$X_1 = C\cos(\theta)$$





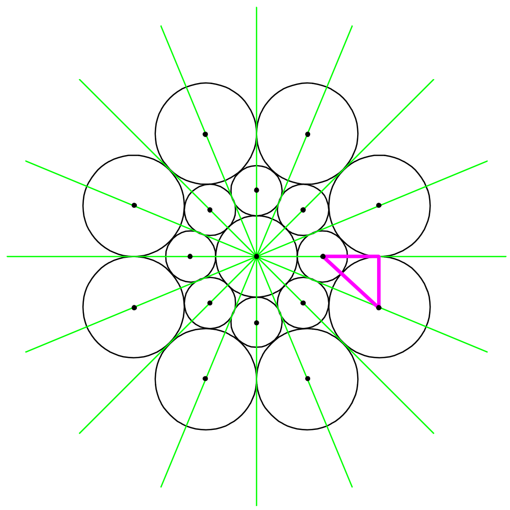
$$\cos(\phi) = \frac{X_1 - B}{X_2}$$

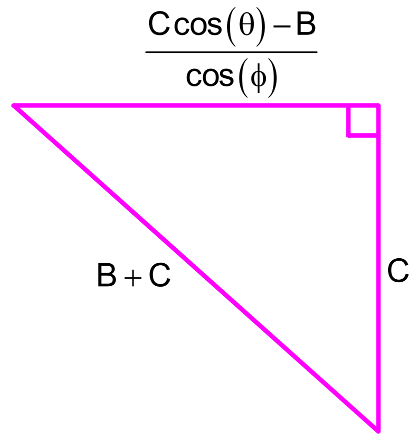
$$X_2 \cos(\phi) = X_1 - B$$

$$X_2 = \frac{X_1 - B}{\cos(\phi)}$$

$$X_2 = \frac{C\cos(\theta) - B}{\cos(\phi)}$$

Using the value of X_2, all three sides of the following triangle can now be defined either by known values or by the value to be calculated, C. The Pythagorean Theorem can be used to define an equation based on these values, and the quadratic formula can then solve for C.





$$(B+C)^2 = C^2 + \left(\frac{C\cos(\theta) - B}{\cos(\phi)}\right)^2$$

$$\cos^2(\phi)(B+C)^2 = C^2\cos^2(\phi) + \left(C\cos(\theta) - B\right)^2$$

$$\cos^2(\phi)\left(B^2 + 2BC + C^2\right) = C^2\cos^2(\phi) + C^2\cos^2(\theta) - 2BC\cos(\theta) + B^2$$

$$B^2\cos^2(\phi) + 2BC\cos^2(\phi) + \cancel{C^2\cos^2(\phi)} = \cancel{C^2\cos^2(\phi)} + C^2\cos^2(\theta) - 2BC\cos(\theta) + B^2$$

$$0 = \left[\cos^2(\theta)\right] \bullet C^2 + \left[-2B\cos(\theta) - 2B\cos^2(\phi)\right] \bullet C + \left[B^2 - B^2\cos^2(\phi)\right]$$

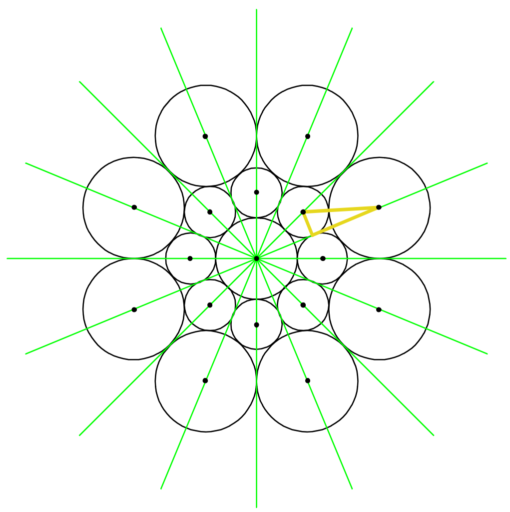
$$0 = \left[\cos^2(\theta)\right] \bullet C^2 + \left[-2B\left(\cos(\theta) + \cos^2(\phi)\right)\right] \bullet C + \left[B^2\left(1 - \cos^2(\phi)\right)\right]$$

$$0 = \underbrace{\left[\cos^2(\theta)\right]}_{a_1} \bullet C^2 + \underbrace{\left[-2B\left(\cos(\theta) + \cos^2(\phi)\right)\right]}_{b_1} \bullet C + \underbrace{\left[B^2\sin^2(\phi)\right]}_{c_1}$$

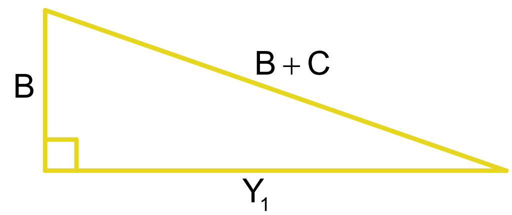
Therefore,

$$C = \frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{2a_1}$$

All that remains is to calculate how far out along each of the rays to place each of the circles. On the following triangle, two sides are known distances, while the third side measures the distance from the point where the ray passes through the tangent point of two circles (which is also the midpoint of the line segment joining the centers of the same two circles) to the point along the ray where the center of the new circle should lie.



$Y_1 =$ The distance between the tangent point and the center



$$B^2 + Y_1^2 = (B+C)^2$$

$$Y_1^2 = (B+C)^2 - B^2$$

$$Y_1 = \sqrt{(B+C)^2 - B^2}$$

Adding a third ring of circles uses exactly the same process, but now it is a new length, D, that is to be calculated while the formula references the old calculation, C. The same holds for the final ring – measurement E can be calculated by referencing known measurement D.

$$0 = \underbrace{\left[\cos^2(\theta)\right]}_{a_2} \bullet D^2 + \underbrace{\left[-2C\left(\cos(\theta) + \cos^2(\phi)\right)\right]}_{b_2} \bullet D + \underbrace{\left[C^2 \sin^2(\phi)\right]}_{c_2}$$

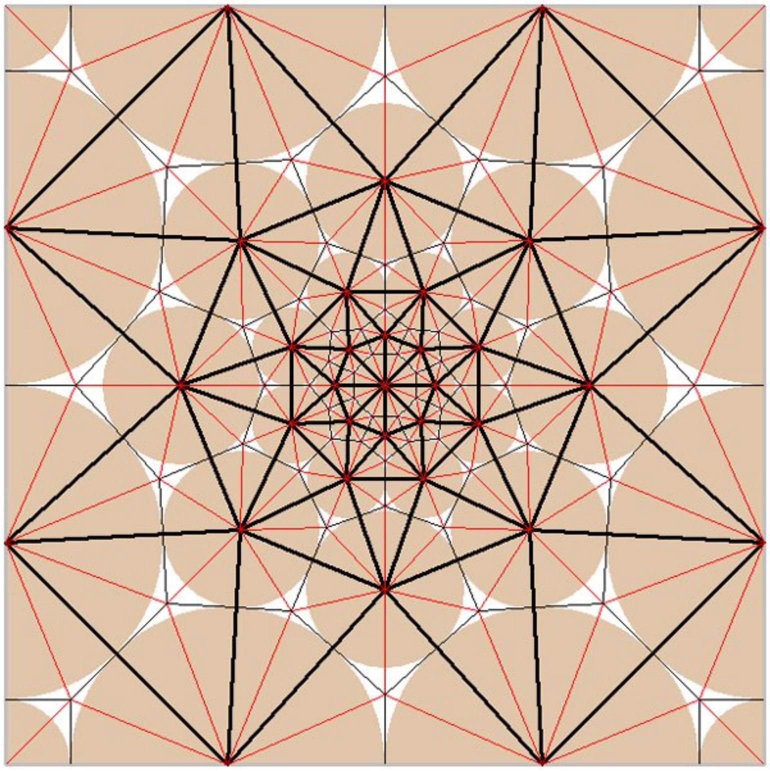
so that $D = \dfrac{-b_2 + \sqrt{b_2^{\,2} - 4a_2c_2}}{2a_2}$ and $Y_2 = \sqrt{(C+D)^2 - C^2}$

and

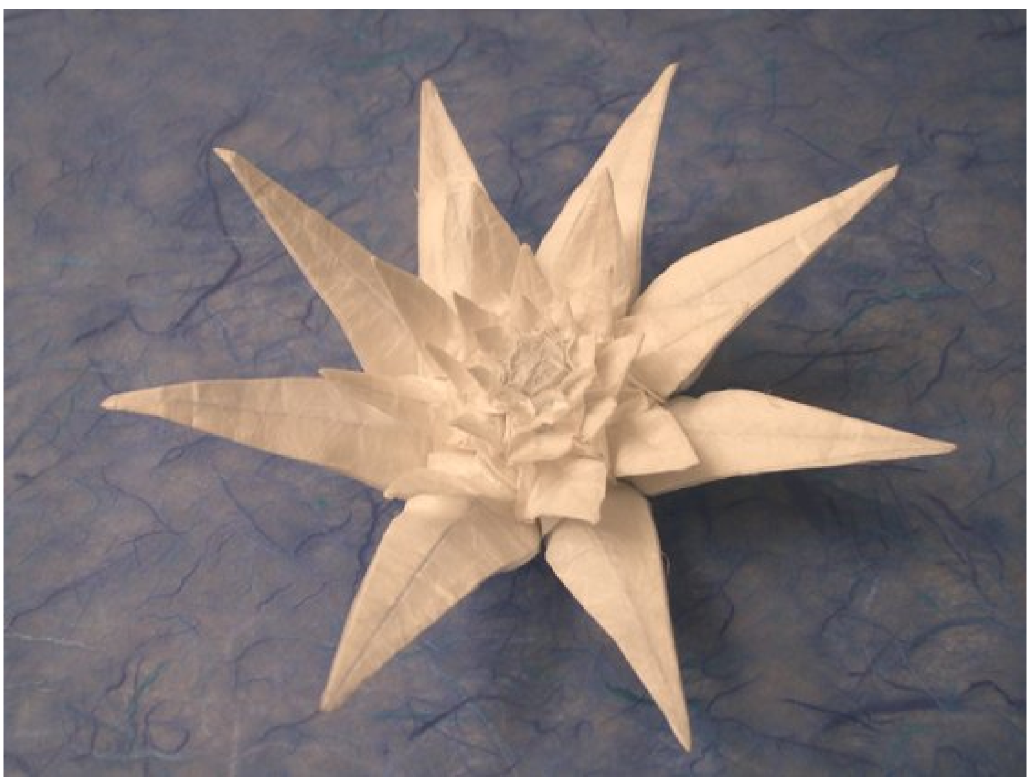
$$0 = \underbrace{\left[\cos^2(\theta)\right]}_{a_3} \bullet E^2 + \underbrace{\left[-2D\left(\cos(\theta) + \cos^2(\phi)\right)\right]}_{b_3} \bullet E + \underbrace{\left[D^2 \sin^2(\phi)\right]}_{c_3}$$

so that $E = \dfrac{-b_3 + \sqrt{b_3^{\,2} - 4a_3c_3}}{2a_3}$ and $Y_3 = \sqrt{(D+E)^2 - D^2}$

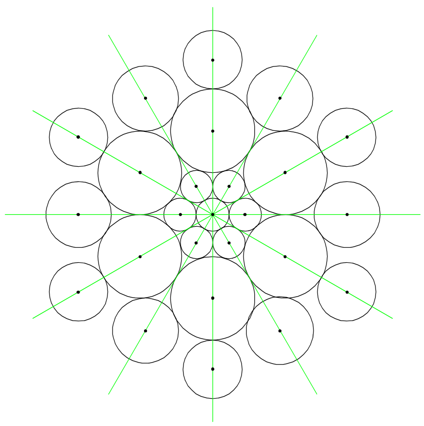
The preceding calculations can be used to create the following crease pattern for my water lily design based solely on the size of the center circle and the number of circles in each ring.

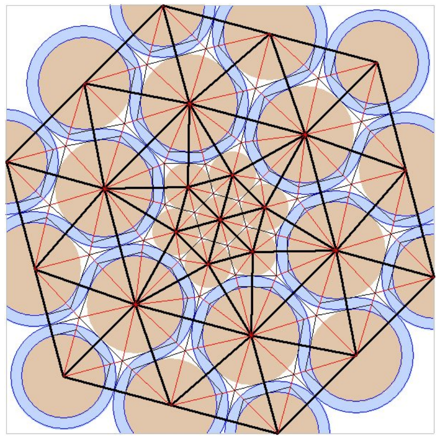


This crease pattern can be used to fold the following origami water lily.

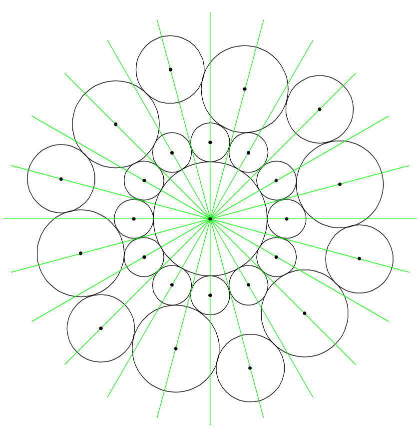


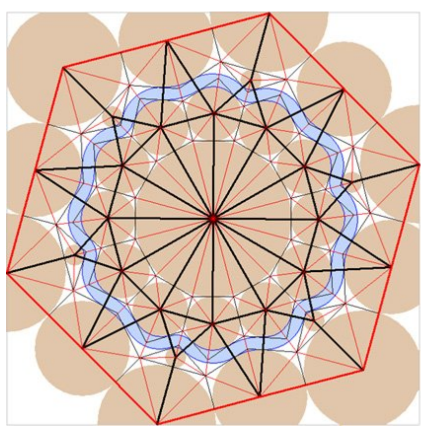
There are many variations of these calculations which can lead to a wide variety of different flowers. The angle θ (and its complementary angle, □) can be changed to give any number of circles in the first ring. Subsequent rings do not necessarily need to be tangent, in which case they can be calculated by multiplying a known value by a scalar. In the following example of my graptopetalum rusbyi design, the central angle is 30° instead of 22.5°, and the radii of the smaller circles in the outer ring are 0.7 times the radius of one of the largest circles in the second ring. The larger circles in the outer ring were sized in order to make the centers line up, creating a regular hexagon.





The torch ginger is my favorite flower as well as my favorite of the origami models I have designed. Although I designed it before the water lily, its crease pattern can be calculated in a similar fashion. The central angle is 15° this time, and the radii of the large circles in the outer ring were originally 2.2 times the radius of one of the small circles in the first ring. The smaller circles in the outer ring are once again positioned and sized in order to create a regular hexagon, although this time they create five-sided molecules. Because of this, extra circles were inserted into the base in order to fill in the crease pattern. I also ended up changing the proportions of the outer ring of circles and adding a river in order to avoid folding gusset molecules.





I have designed and folded more flowers than are shown in this article. For a comprehensive list, with pictures (both of the folded models and of the real objects I was trying to recreate), visit my website at:

http://www.derek.mcgann.com/