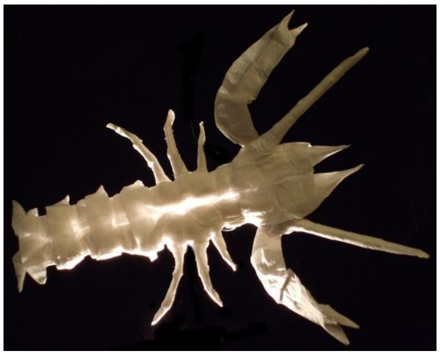
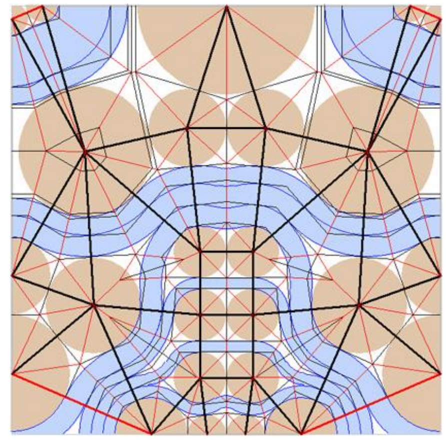
A General Approach to Crease Pattern Construction

by Derek McGann

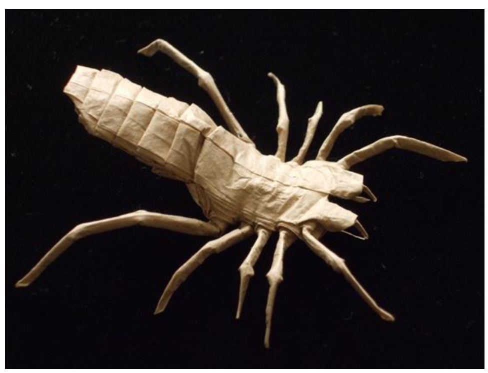
While flowers have always been, and will probably always be, my favorite origami subject (both to fold and to design), there are other types of models that I enjoy creating as well. The methods outlined in my article "Origami Flower Design" were the result of my desire to accurately construct crease patterns based on mathematical calculations – arranging the circles freehand invariably results in a small amount of error which propagates as the design becomes more complicated. Since my flowers relied on radial symmetry, creating a series of right triangles radiating from the center, my previous method was too specific - I needed a similar method which was more generalized for any type of crease pattern I might wish to create. The first design for which this became a serious issue was my

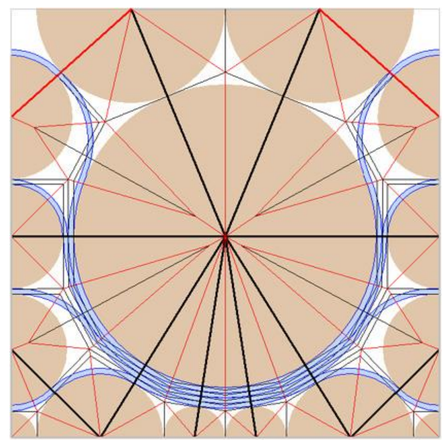


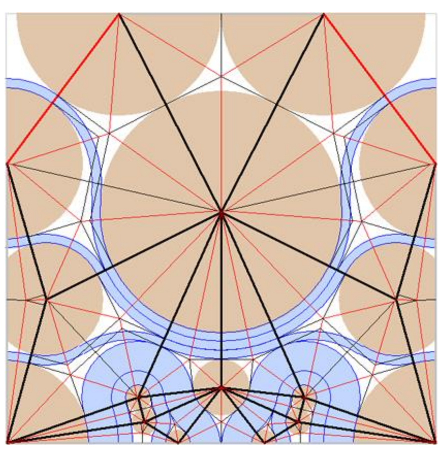
Ghost Shrimp design. This crease pattern was too complex for the propagated error resulting from a freehand arrangement to be acceptable. It was at this point that I completely developed a more general calculation approach. I found that this approach had other advantages, so I began using it for all my designs. Shown below are two notable examples - my Camel



Spider II and Whip Scorpion. The Camel Spider crease pattern has a perfectly horizontal axial crease running through it. The original circle arrangement had the centers of the two small circles out-of-line with the large circle – since the crease pattern was calculated from pre-determined flap lengths, I was able to make minute adjustments to those

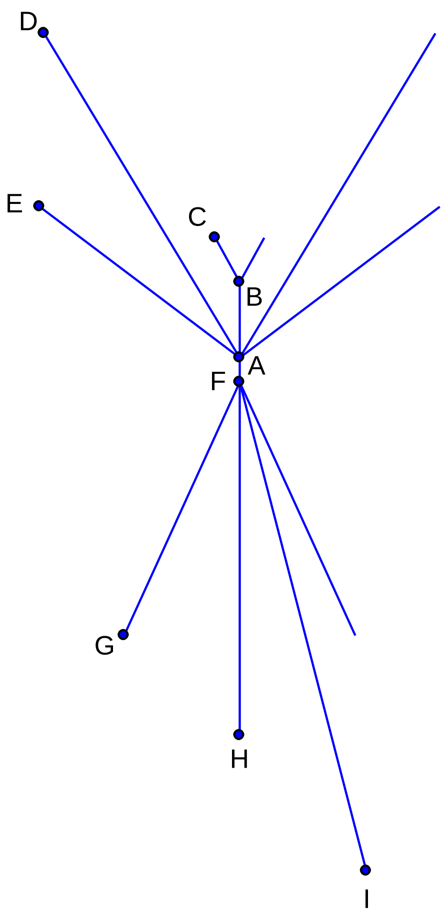






flap lengths in order to line the circles up, simplifying the design. The Whip Scorpion crease pattern was originally not quite square. Again, by making small adjustments to the original flap lengths (small enough to not effect the aesthetic appearance of the final model), I was easily able to make it fit into a perfect square. Since both designs were constructed from precise mathematical calculations (using Geometer's Sketchpad), the crease patterns **automatically adjusted themselves** when I made the previously mentioned minor adjustments. Even though it takes a little more time at the beginning of the design process, I have found that the ability to make those adjustments has saved me much time in the long run.

Since developing that process, I have gone back and re-folded some of my previous designs in order to improve them. I also re-constructed the crease patterns using my newly developed technique. One such re-done older design is my Walking Leaf – this model will serve as a perfect example of this general calculation technique because it is the simplest model I have designed which encompasses all the variations I have currently encountered of this process. The first step is to define a stick-figure which represents the desired subject. Below is my Walking Leaf stick figure.



The following line segments represent the various parts of the insect I was trying to create. The circle-river packing function is also indicated (flap or river) as well as the proportional length.

$\overline{AB}$ = Neck (River): 0.15

$\overline{BC}$ = Antennae (Flaps): 0.10

$\overline{AD}$ = Forelegs (Flaps): 0.75

$\overline{AE}$ = Midlegs (Flaps): 0.50

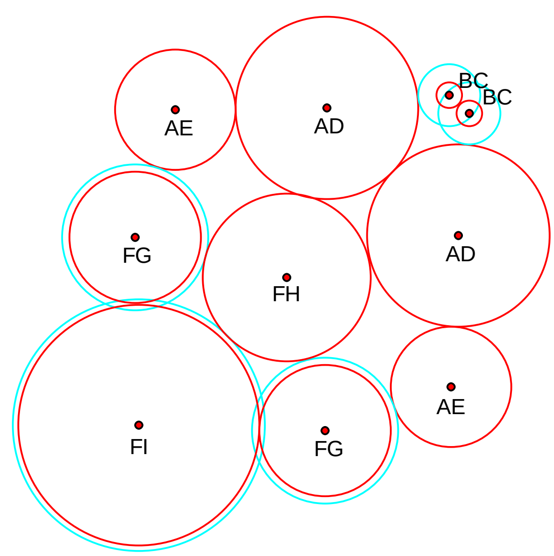
$\overline{AF}$ = Aesthetic Simplification (River): 0.05

$\overline{FG}$ = Hindlegs (Flaps): 0.55

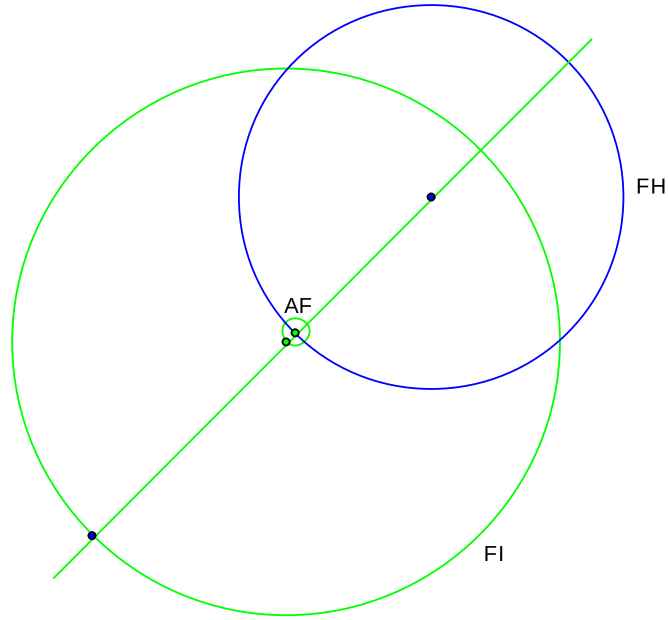
$\overline{FH}$ = Leafy Abdomen (Flap): 0.70

$\overline{FI}$ = Leafy Abdomen II (Flap): 1.00

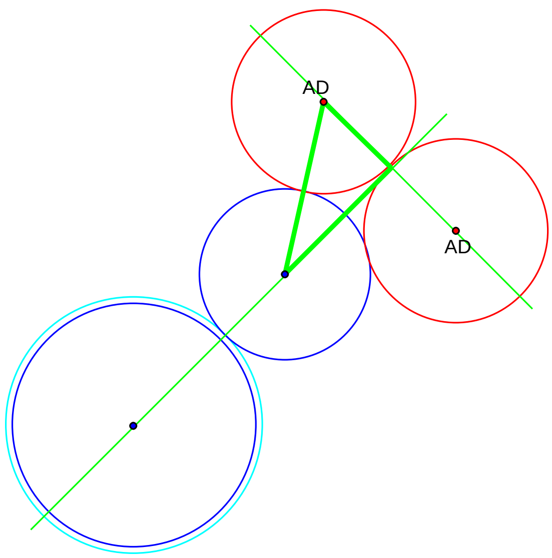
The "aesthetic simplification" is a river I added to the crease pattern **after** folding it for the first time – it was not part of my original design. I wanted both "leafy abdomens" to be smooth and unbroken – the original design had a gusset molecule in the region of "leafy abdomen II". I stretched that part out to make it more aesthetically pleasing and later realized what I had done. I had inadvertently added a river and converted the gusset molecule to two rabbit-ear molecules (see the Walking Leaf crease pattern at the end of this article for the end result). The second step is to convert this stick figure to a set of circles and "rivers". The circles are named by the segment length that their radius represents (circle BC has a radius the same length as $\overline{BC}$). "Rivers" are not originally represented as such. Since important information about the rivers is not known at the beginning of the design process (the length of the rivers, the number and location of the "bends" in the rivers), I initially represent rivers by adding concentric circles to the flaps affected by them. At this point, I also choose a symmetry and arrange the circles roughly into a square. The trick is to arrange them so that they have the correct positioning and layering in relation to each other. In this case, the result is obviously not square, but it is reasonably close. Any adjustments to the proportional lengths of the flaps would result in a need to rearrange the circles to once again be tangent.

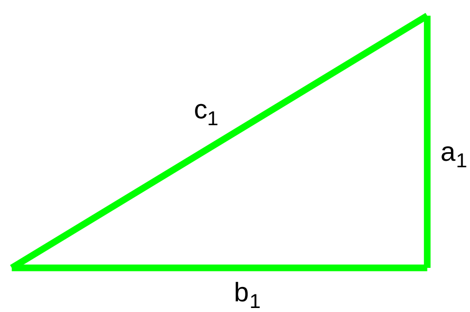


Now the precise construction of the crease pattern based on mathematical calculations begins. I chose circle FH to be the main flap which the rest of the crease pattern is arranged around. Since circle FI is also on the line of symmetry (in this case, diagonal symmetry), its center is easy to construct. At the intersection of circle FH and the line of symmetry, a circle of radius AF is constructed, representing the river. Where that circle intersects the line of symmetry, a circle of radius FI is constructed. This circle intersects the line of symmetry where the center of flap FI should lie. This process will be used later in the construction process whenever it becomes necessary to locate the center of a flap when a line passing through its center has already been constructed.



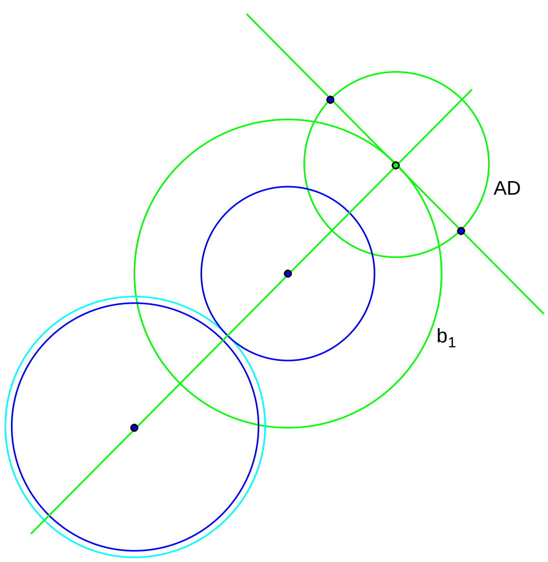
Next, circles AD are added to the crease pattern. Since they are tangent to the line of symmetry, their position can be calculated as follows, using the Pythagorean Theorem:





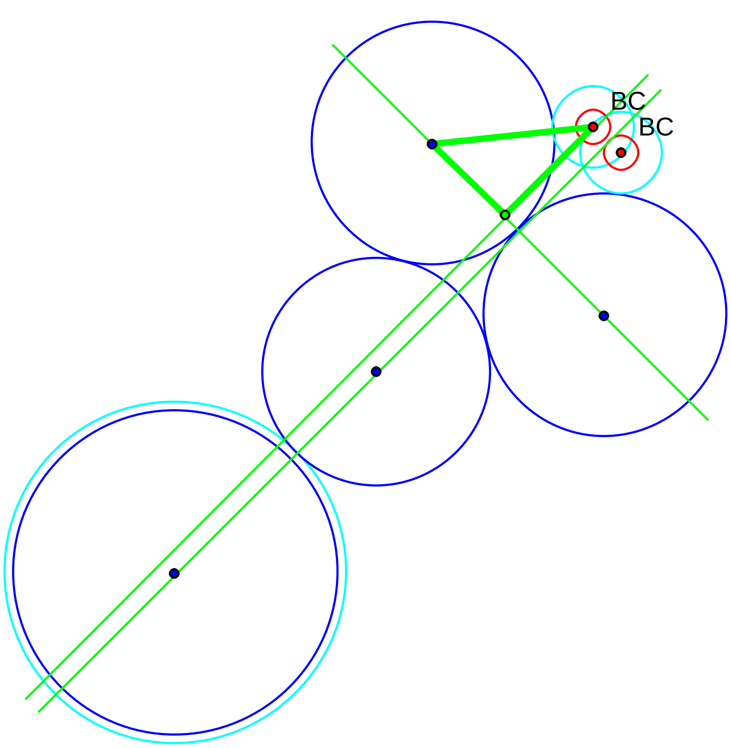
$$a_1 = \overline{AD}$$
$$b_1 = ?$$
$$c_1 = \overline{AD} + \overline{FH}$$

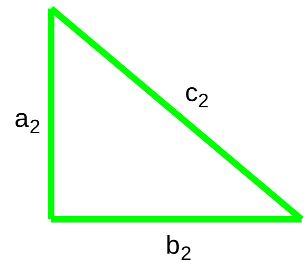
$$a_1^2 + b_1^2 = c_1^2$$
$$b_1^2 = c_1^2 - a_1^2$$
$$b_1 = \sqrt{c_1^2 - a_1^2}$$



Calculation b_1 can now be used to locate the line which is perpendicular to the line of symmetry along which the centers of circles AD lie. The point where this line hits the line of symmetry, as well as where along this line the centers of AD should lie, can be constructed as already described for circle FI.

Circles BC are now added to the crease pattern using the same process. The only difference is that a line parallel to the line of symmetry is constructed through the center of circle BC.

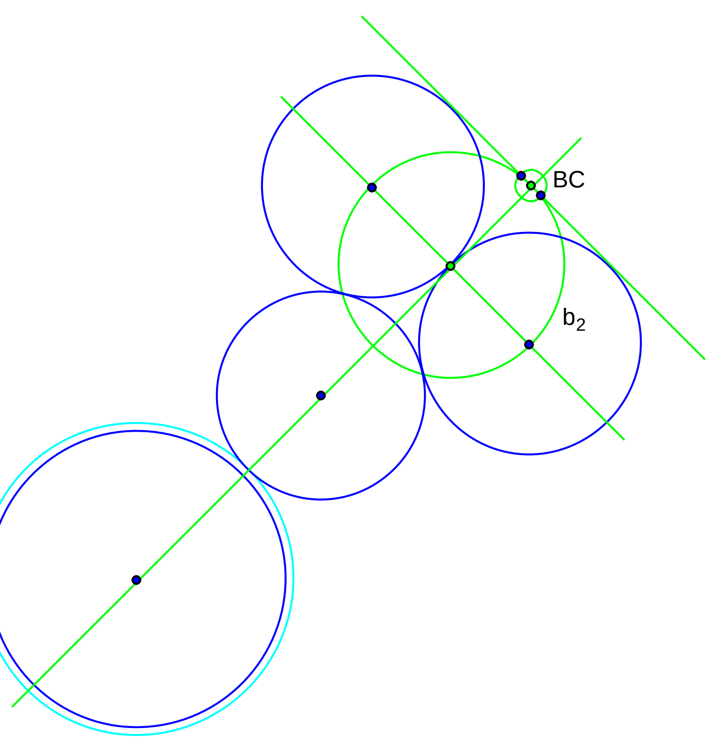




$$a_2 = \overline{AD} - \overline{BC}$$
$$b_2 = ?$$
$$c_2 = \overline{AD} + \overline{AB} + \overline{BC}$$

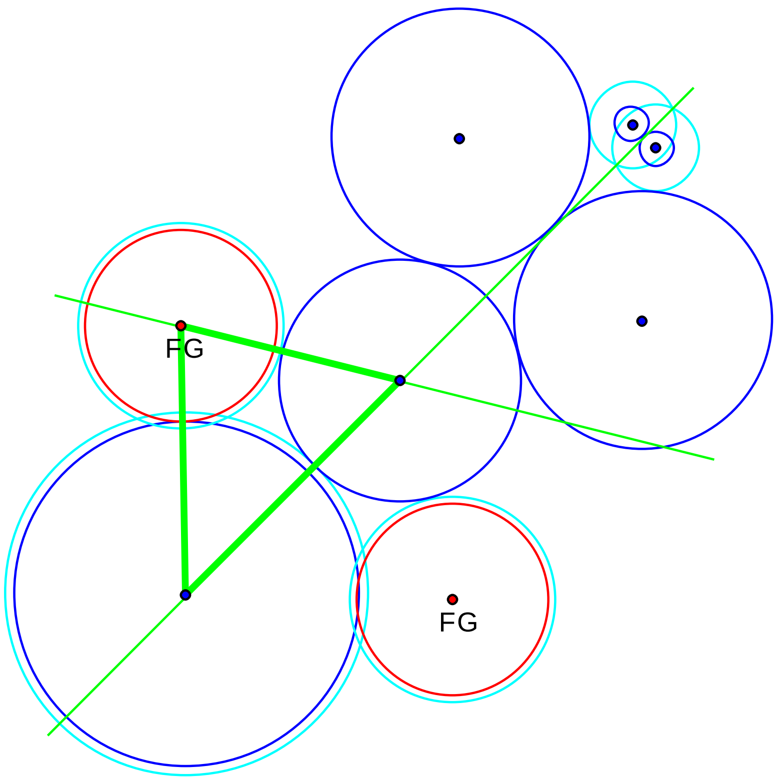
$$a_2^{\ 2} + b_2^{\ 2} = c_2^{\ 2}$$
$$b_2^{\ 2} = c_2^{\ 2} - a_2^{\ 2}$$
$$b_2 = \sqrt{c_2^{\ 2} - a_2^{\ 2}}$$

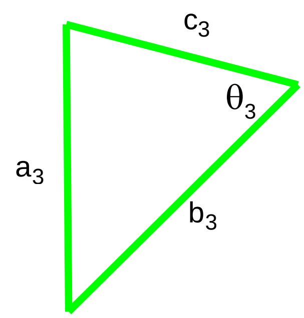
Once again, calculation b_2, along with the value of flap BC, can be used to construct the positions of the centers for circles BC.



Since these triangles have all been right triangles, these calculations have all been similar to those outlined in my previous article, “Origami Flower Design”. Since the rest of the flaps do not have centers on, and are not tangent to, the line of symmetry, the rest of the construction process will be distinctly different. I will only demonstrate the construction process for one of each pair of circles – the other will be constructed by a reflection over the line of symmetry.

The positions of circles AE are determined by circles FG and AD. Because of this, circles FG must be added next using a similar process. Since this is a non-right triangle, the line which passes through the center of circle FG will be constructed by rotating the line of symmetry around the center of circle FH. Since all three sides of the triangle are known values, this angle can be calculated by using the Law of Cosines. Since the rotation is clockwise, the angle should be negative.





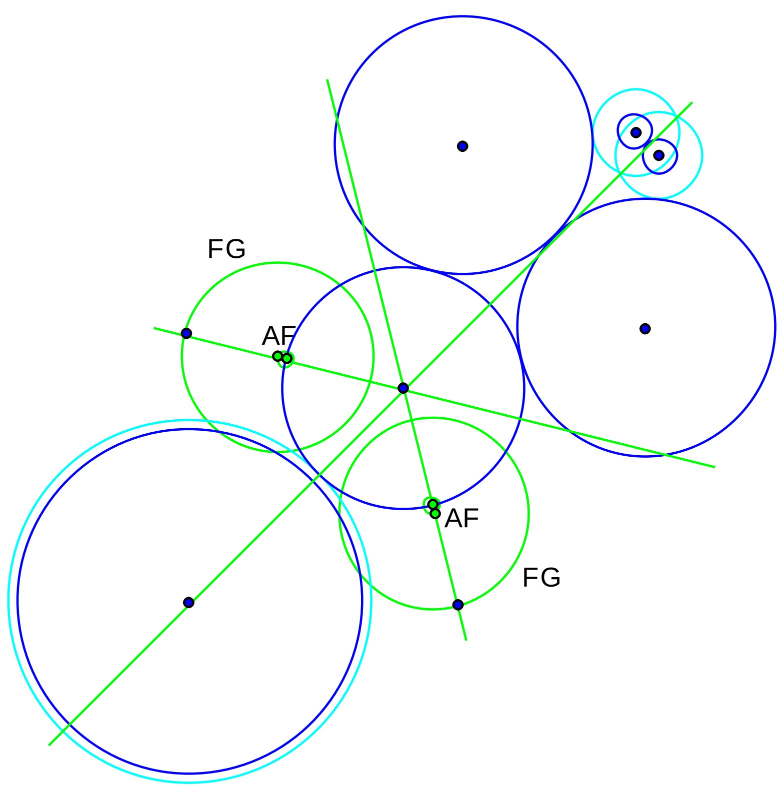
$$a_3 = \overline{FI} + \overline{FG}$$

$$b_3 = \overline{FI} + \overline{AF} + \overline{FH}$$

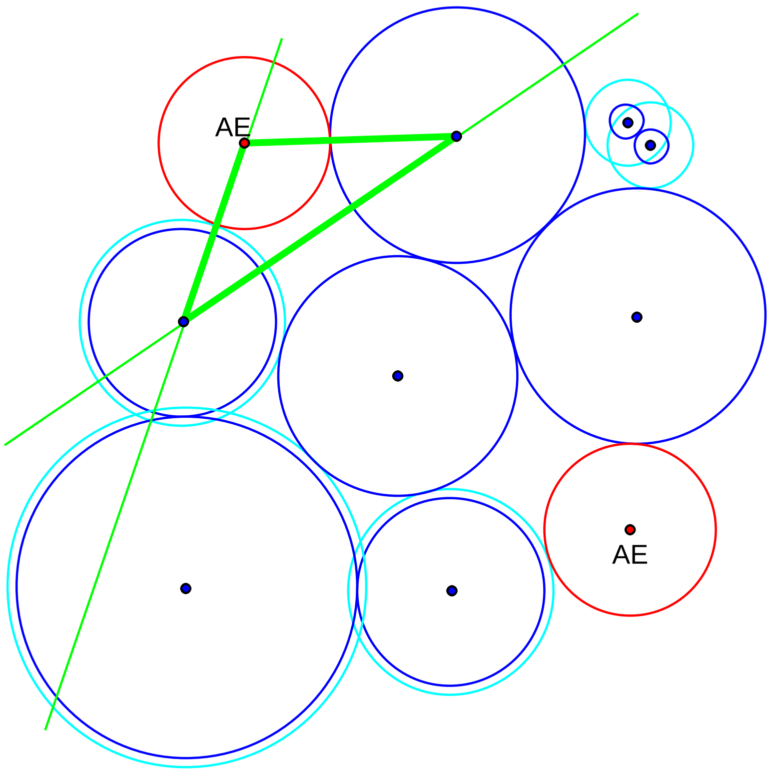
$$c_3 = \overline{FH} + \overline{AF} + \overline{FG}$$

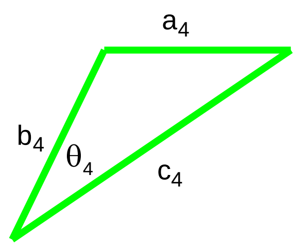
$$\theta_3 = -\arccos\left(\frac{b_3{}^2 + c_3{}^2 - a_3{}^2}{2b_3c_3}\right)$$

Now that this line is constructed, the values of AF and FG are all that are necessary to locate the centers of circles FG.



The last parts of the arrangement that need to be added are circles AE. At first glance, it seems that this calculation will be more difficult because these circles complete four sided molecules. A triangle can still be constructed, however. Since the endpoints of side c_4 are constructed points, the length of this segment is a set value that can be measured.

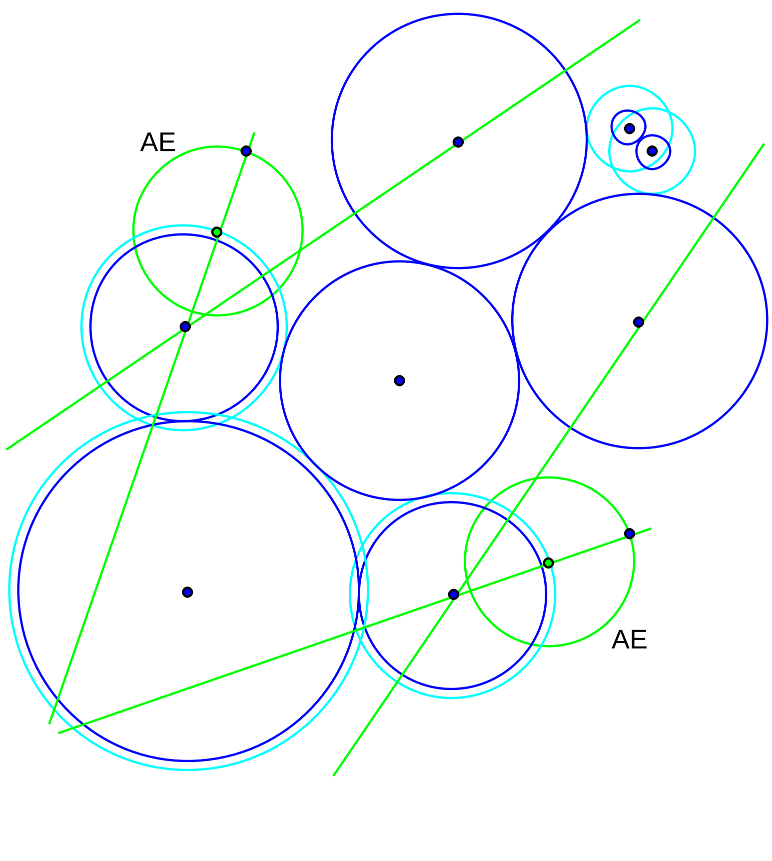




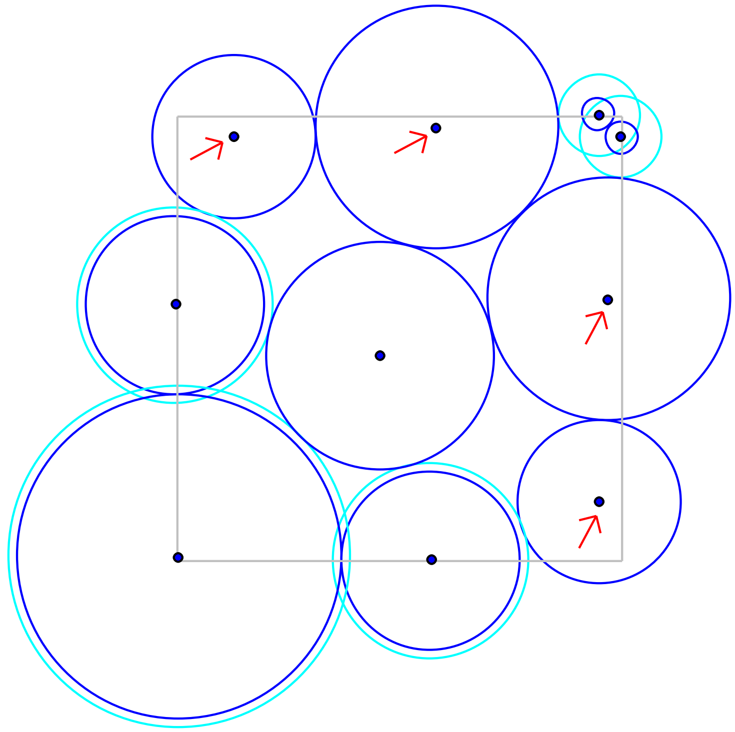
$$a_4 = \overline{AE} + \overline{AD}$$
$$b_4 = \overline{FG} + \overline{AF} + \overline{AE}$$
$$c_4 = \text{measured value}$$

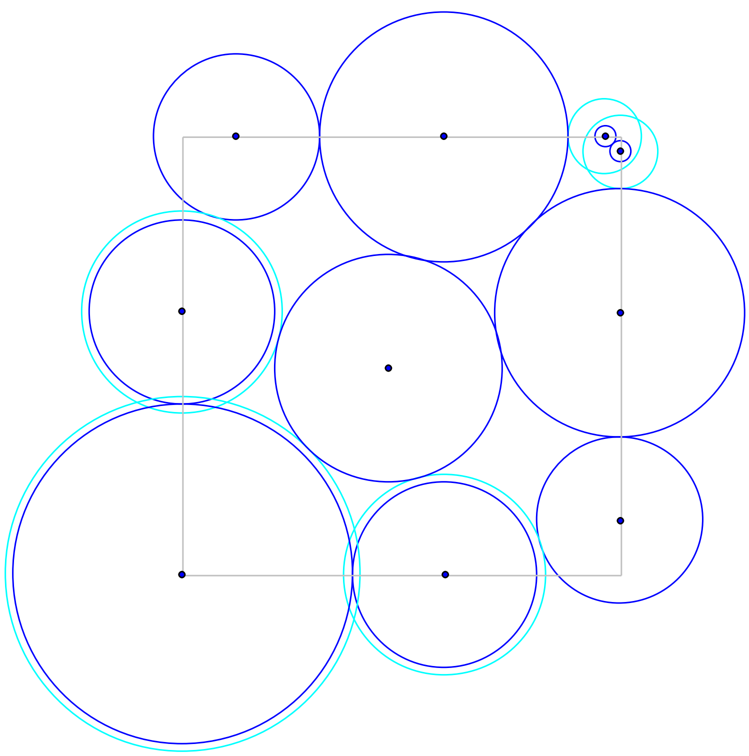
$$\theta_4 = \arccos\left(\frac{b_4^{\,2} + c_4^{\,2} - a_4^{\,2}}{2b_4c_4}\right)$$

Once the line passing through the two already-constructed circle centers is rotated by angle θ_4, the centers of circles AE can be constructed by simply using the value of AE.



The circle-river packing is now constructed, and it is at this point that the benefit of this construction method is evident. When the arrangement is placed within a square, it is clear that with a few minor alterations, four of the circles that are currently slightly in the interior of the paper can be moved to the edge. Since the entire structure is constructed from the proportional flap lengths given at the beginning of this article, any alterations to those flap lengths will change the sizes of the circles. However, because of that construction, they will **automatically remain tangent** to each other at the desired points. Obviously, major alterations to the flap proportions can lead to inconsistencies which will cause some, if not all, of the constructed pattern to disappear, but minor adjustments are now simple to experiment with.





Below are the proportions used in the final folded version of my Walking Leaf.

$\overline{AB}$ = Neck (River): 0.154

$\overline{BC}$ = Antennae (Flaps): 0.063

$\overline{AD}$ = Forelegs (Flaps): 0.734

$\overline{AE}$ = Midlegs (Flaps): 0.489

$\overline{AF}$ = Aesthetic Simplification (River): 0.048

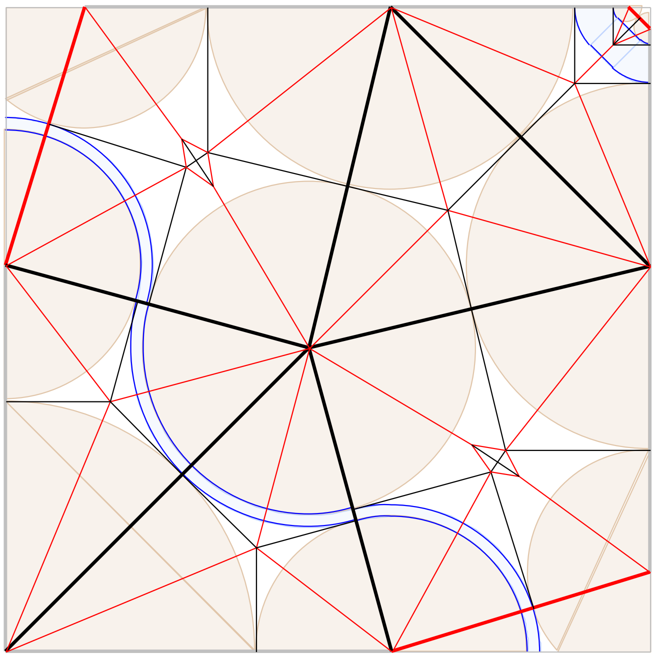
$\overline{FG}$ = Hindlegs (Flaps): 0.545

$\overline{FH}$ = Leafy Abdomen (Flap): 0.669

$\overline{FI}$ = Leafy Abdomen II (Flap): 1.00

These extremely small alterations allowed all of the centers of the outer circles to lie on either the corners or the edges of the paper. Anyone familiar with the details of technical origami design will understand the benefit of this arrangement.

At this point, the axial folds of the crease pattern can be constructed between the centers of tangent circles, and the crease patterns of these molecules can be individually constructed. Below are the final crease pattern and folded model of my Walking Leaf.



This is the construction method I currently use for all of my original designs. For a complete list of those designs, along with pictures of the constructed base crease patterns and the finished models, visit my website:

http://www.derek.mcgann.com/